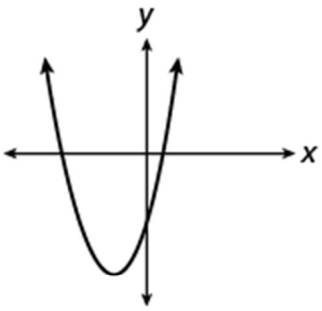
Quadratic Equations, Imaginary Numbers, Practice **Solutions**

1.



Which of the following equations could represent the graph in the figure?

Ⓐ $y = x^2 - 4x - 4$
Ⓑ $y = x^2 + 4x - 4$
Ⓒ $y = x^2 - 8x + 16$
Ⓓ $y = x^2 + 8x + 16$

Difficulty: Medium

Category: Passport to Advanced Math / Quadratics

Strategic Advice: Factoring the quadratic equations could give you information about the *x*-intercepts, but upon inspection, A and (B) can't be factored. As an alternate strategy, find the axis of symmetry using the formula $x = -\frac{b}{2a}$ (the quadratic formula without the radical part) to determine in which quadrant the vertex lies. You are looking for an equation whose graph has its vertex in the third quadrant.

Getting to the Answer: Choice (B) is correct because $x = -\frac{(4)}{2(1)} = -\frac{4}{2} = -2$, and when −2 is substituted into the equation $y = (-2)^2 + 4(-2) - 4 = -8$, it puts the vertex at (−2, −8), which is in the third quadrant and matches the graph.

2.

$$\frac{2}{i+6} + (2+5i)$$

Which of the following expressions is equivalent to the complex number given above?

Note that $\sqrt{-1} = i$.

Ⓐ $\frac{32i+9}{i+6}$

Ⓑ $\frac{34i+7}{i+6}$

Ⓒ $\frac{32i+19}{i+6}$

Ⓓ $\frac{37i+14}{i+6}$

Difficulty: Hard

Category: Additional Topics in Math / Imaginary Numbers

Strategic Advice: Fractions with complex numbers are no different from any other fraction. You must find a common denominator before adding them.

Getting to the Answer: Find a common denominator by multiplying the second term by $i + 6$. You're given that $\sqrt{-1} = i$, but a more useful fact is that $i^2 = -1$, so be sure to make this substitution as you go. Once you have found the common denominator, you can simply add like terms.

$$\begin{aligned}\frac{2}{i+6}+(2+5i) &= \frac{2}{i+6}+\frac{2+5i}{1}\\ &= \frac{2}{i+6}+\frac{2+5i}{1}\left(\frac{i+6}{i+6}\right)\\ &= \frac{2}{i+6}+\frac{2i+12+5(-1)+30i}{i+6}\\ &= \frac{2}{i+6}+\frac{32i+7}{i+6}\\ &= \frac{32i+9}{i+6}\end{aligned}$$

3. Calculator

Given that $\sqrt{-1} = i$, which of the following is equivalent to the sum $i^{125} + i^{125}$?

Ⓐ i^{14}
Ⓑ i^{250}
Ⓒ $2i^{45}$
Ⓓ $2i^{250}$

Difficulty: Medium

Category: Additional Topics in Math / Imaginary Numbers

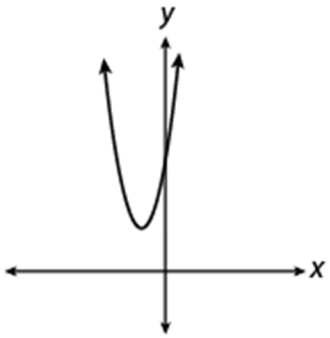
Strategic Advice: To evaluate a high power of i, look for patterns and use the definition $\sqrt{-1} = i$, which, when written in a more useful form, is $i^2 = -1$.

Getting to the Answer: Write out enough powers of i for you to see the pattern:

$i^1 = i$
$i^2 = -1$ (by definition)
$i^3 = i \times i^2 = i \times -1 = -i$
$i^4 = i^2 \times i^2 = -1 \times -1 = 1$
$i^5 = i^4 \times i = 1 \times i = i$
$i^6 = i^4 \times i^2 = 1 \times -1 = -1$
$i^7 = i^6 \times i = -1 \times i = -i$
$i^8 = i^4 \times i^4 = 1 \times 1 = 1$

Notice that the pattern (i, -1, $-i$, 1, i, -1, $-i$, 1) repeats on a cycle of 4. To evaluate i^{125}, divide 125 by 4. The result is 31 remainder 1, which means 31 full cycles, and then back to i^1. This means i^{125} is equivalent to i^1, which is i. Because $i + i = 2i$, you are looking for the answer choice that is also equivalent to $2i$. Choices (C) and D look tempting (because of the 2), so start with them: (C) is correct because $45 \div 4 = 11$, remainder 1, which means i^{45} is equivalent to i and $2i^{45}$ is equal to $2i$.

4. Calculator



If the equation of the graph shown above is $y = 2(x + 3)^2 + 10$, what is the y-intercept of the graph?

Category: Passport to Advanced Math / Quadratics

Strategic Advice: Graphically, a y-intercept is in the form $(0, y)$, so the y-intercept of the graph is the value of y when 0 is substituted for x in the equation.

Getting to the Answer: Don't forget to follow the correct order of operations as you simplify the expression.

$y = 2(0 + 3)^2 + 10$

$= 2(3)^2 + 10$

$= 2(9) + 10$

$= 18 + 10$

$= 28$

5.

If $x^2 - 8x = 48$ and $x < 0$, what is the value of $x + 10$?

Ⓐ −2
Ⓑ 4
Ⓒ 6
Ⓓ 8

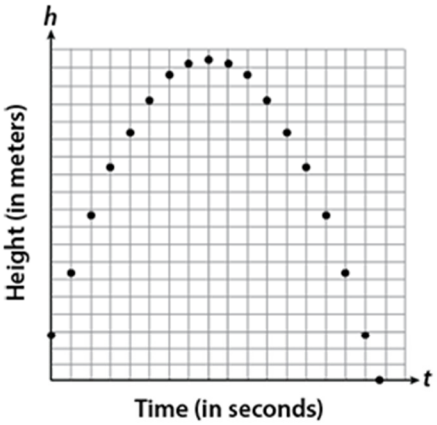
Difficulty: Easy

Category: Passport to Advanced Math / Quadratics

Strategic Advice: There are a number of ways to solve quadratic equations. When the coefficient of x^2 is 1, the quickest way is usually to factor, if possible. You could also use the quadratic formula or completing the square.

Getting to the Answer: To answer this question, first rewrite the equation in standard form, $x^2 - 8x - 48 = 0$, and then factor to arrive at $(x - 12)(x + 4) = 0$. Using the Zero-Product property to solve for x results in $x = 12$ and $x = -4$. It is given that $x < 0$, so x must equal -4. This means that $x + 10$ is equal to $-4 + 10 = 6$.

6.



A physics class is using simulation software to study water bottle rockets before attempting to build one for the National Physics Competition. Their first simulation is of a rocket without a parachute launched from the roof of the gymnasium. The scatterplot shows the approximate path of the rocket. The software program generates the equation $h = -4.9t^2 + 39.2t + 14$ to model the data, where h is the height in meters of the rocket t seconds after it was launched. What does the number 14 most likely represent in this equation?

Ⓐ The number of seconds the rocket was in the air
Ⓑ The height of the gymnasium from which the rocket was launched
Ⓒ The number of seconds that it took the rocket to reach its maximum height
Ⓓ The maximum height of the rocket

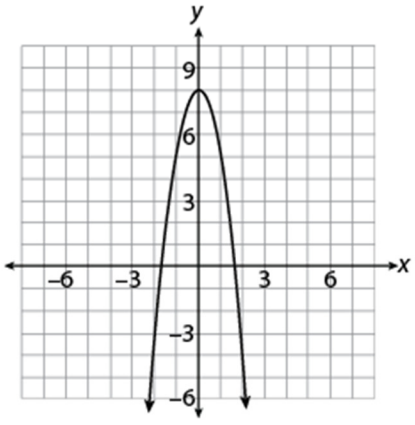
Difficulty: Medium

Category: Passport to Advanced Math / Quadratics

Strategic Advice: When a quadratic equation is written in standard form, $y = ax^2 + bx + c$, the value of c is the y-intercept of the equation's graph. This is because substituting 0 for x results in $y = a(0)^2 + b(0) + c = c$.

Getting to the Answer: In a real-world scenario, the y-intercept represents an initial amount. In this question, height is what is being measured, so the y-intercept represents the initial height of the bottle rocket. Because the rocket was fired from the roof of the gymnasium, the height of the gymnasium must be 14 meters, making (B) correct.

7.



Vadim graphs the equation $y = -3x^2 + 8$, which is shown in the figure above. He realizes, however, that he miscalculated and should have graphed $y = -\frac{1}{3}x^2 + 8$. How will this affect his graph?

Ⓐ It will change the y-intercept.
Ⓑ It will make the parabola open in the opposite direction.
Ⓒ It will make the parabola cross the x-axis closer to the origin.
Ⓓ It will make the parabola cross the x-axis farther from the origin.

Difficulty: Medium

Category: Passport to Advanced Math / Quadratics

Strategic Advice: Don't waste time trying to graph the second equation. Instead, think about the question conceptually. The magnitude of the coefficient of x^2 (not the sign) determines how wide or narrow the graph is.

Getting to the Answer: Changing the coefficient of x^2 from –3 to $-\frac{1}{3}$ will make the graph narrower or wider (in this case, wider), which means the only things that will change are the x-intercepts. This means you can eliminate A and B. To choose between C and (D), recall that fraction coefficients (between 0 and 1) result in wider graphs, so the x-intercepts will spread out and therefore be farther from the origin.

8.

x	1	2	3	4	5	6
$f(x)$	3.5	0	–2.5	–4	–4.5	–4

The table above shows several points through which the graph of a quadratic function $f(x)$ passes. One of the x-intercepts for the graph is given in the table. What is the other x-intercept for the graph?

Ⓐ (–2, 0)
Ⓑ (5, 0)
Ⓒ (8, 0)
Ⓓ (10, 0)

Difficulty: Medium

Category: Passport to Advanced Math / Quadratics

Strategic Advice: This question is much simpler than it looks. Don't waste time trying to find the equation of the quadratic. Rather, think about properties of parabolas, in particular, symmetry.

Getting to the Answer: The graph of a parabola is symmetric with respect to its axis of symmetry (the imaginary vertical line that passes through the x-coordinate of the vertex). This means that each x-intercept must be the same distance from the vertex. Take a careful look at the values in the table. The y-values start at 3.5, decrease to a minimum value of –4.5, and then turn around. The points on each side of the minimum have the same y-values (–4), which means you've found the vertex, (5, –4.5). The x-intercept given in the table is (2, 0), which is 3 horizontal units to the left of 5. Therefore, the other x-intercept must be 3 horizontal units to the right of 5, which is (8, 0).